CERTIFIED PUBLIC ACCOUNT

FOUNDATION LEVEL 1 EXAMINATION

F1.1: BUSINESS MATHEMATICS AND QUANTITATIVE METHODS

MODEL ANSWER AND MARKING GUIDE

QUESTION ONE **Mark(s)**

Marking Scheme

a)	Define single price index	1
	Define composite price index	1
	Maximum	**2**
b)	Arithmetic mean of first part (2005-2007)	1
	Arithmetic mean of first part (2009-2011)	1
	Not considering year 2008	0.5
	Conclusion	0.5
	Maximum	**3**
c)	∑ cost of (A&B) at base quantity and current prices (p_1q_0)	0.5
	∑ cost of (A&B) at base quantity and base period prices (p0q0)	0.5
	∑ cost of (A&B) at current quantity and base period prices (p_0q_1)	0.5
	∑ cost of (A&B) at current quantity and current period prices (p_1q_1)	0.5
	Using data in Laspeyre's index formula	1
	Using data in Paasche's index formula	1
	Computation of value of X	2
	Maximum	**6**
d)	(i) 5-month Moving Average for 45-month	1
	5-month Moving Average for 46 month	1
	5-month Moving Average for 47 month	1
	5-month Moving Average for 48 month	1
	5-month Moving Average for 45 month and estimate for 52 month	1
	Maximum	**5**
	(ii) Value for Month 51	1
	Value for Month 50	1
	Value for Month 49	1
	Weighted three-month average	1
	Maximum	**4**
	Total Marks	**20**

Detailed answer

a) The single price index measures the percentage change in the current price per unit of a product to its base period price, whereas a composite price index measures the average price change for a group of items taken as a whole from a base period to the current period.

b)

Year (y)	*Units Sold (x)*	
2005	100	
2006	120	(100+120+95)/3 =315/3 = 105
2007	95	
2008	105	
2009	108	

2010	102	(108+102+112)/3 = 322/3 = 107.33
2011	112	

Thus, we get two points 105 and 107.33 which we shall plot corresponding to the years 2006 and 2010. By joining these points, we will obtain the required trend line.

c)

Item	p_0	q_0	p_1	q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	1	10	2	5	20	10	10	5
B	1	5	x	2	5x	5	2x	2
					20+5x	15	10+2x	7

Laspeyre's index = $\frac{\sum p_1q_0}{\sum p_0q_0} = \frac{20+5x}{15}$

Paasche's index = $\frac{\sum p_1q_1}{\sum p_0q_1} = \frac{10+2x}{7}$

Given $\frac{(20+5x)/15}{(10+2x)/7} = \frac{28}{27}$

Or $\frac{20 + 5x}{15} \times \frac{7}{10+2x} = \frac{28}{27}$ or x=4

d) (i) Calculations for five-month moving average are shown in table below

Month	Actual demand	5-month Moving Total	5-month Moving Average
43	105	-	-
44	106	-	-
45	110	545	109.50
46	110	561	112.2
47	114	585	117.0
48	121	603	120.6
49	130	630	126.0
50	128	-	-
51	137	-	-

The five-month average demand for month 52 is

$$\frac{\sum x}{\text{Number of period}} = \frac{114+121+130+128+137}{5} = 126 \text{ units}$$

(ii) Weighted three-month average as per weights is as follows:

$$\bar{x}_{weighted} = \frac{\sum \text{weight x Data value}}{\sum \text{Weight}}$$

where

Month	Weight	x value	= Total
51	3	x 137	= 411
50	2	x 128	= 256
49	1	x 130	= 130
	6		797

$$\bar{x}_{weighted} = \frac{797}{6} = 133 \text{ units}$$

QUESTION TWO

Marking Scheme

a)	Advantage (1 Mark for 3correct answer, maximum 3)	3
	Maximum Marks	**3**
b)	Expected time and variance for each activity (0.5 Marks for correct answer, **maximum 5)**	5
i	Draw network (0.5Marks for each correct drawn activity, maximum 5)	5
ii)	Identify critical path and project duration	**2**
iii)	Calculation of value of Z	1
	The probability of completing the project within19 days	1
iv)	iv) Calculation of value of Z	1
	The probability of completing the project within24 days	1
	Conclusion	1
	Maximum Marks	17
	Total	**20**

Detailed answer

a) For many years now, PERT/CPM has become highly computerized. There has been a remarkable growth in the number and power of software packages for PERT/CPM that run on personal computers or workstations. Project management software (for example. Microsoft Project) is a standard tool for project managers although there are dozens of software packages for project management).This has enabled applications to numerous projects each involving many millions of dollars and thousands of activities. Possible revisions in the project plan now can be investigated almost instantaneously. Actual changes and the resulting updates in the schedule, etc. are recorded virtually effortlessly Communications to all parties involved through computer networks and telecommunication systems also have become quick and easy

b)

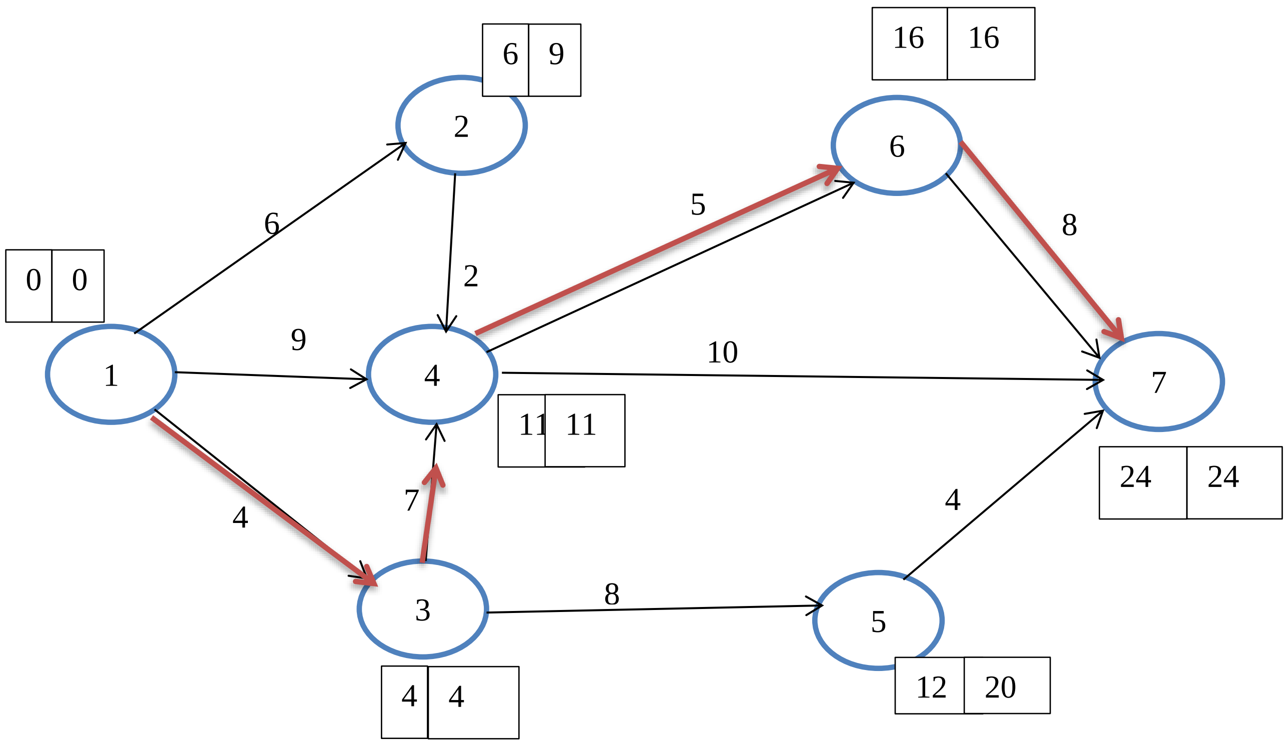
(i) We first compute the time expected for each activity t_a and the variance of each activity to get the following table

Time expected for each activity $t_a = \frac{t_0+4t_m+t\_p}{6} = \frac{4+4(6)+8}{6} = \frac{36}{6}$ =6 days for activity 1-2

The variance of activity is calculated using the formula $\sigma^2 = \left(\frac{t_p-t_0}{6}\right)^2 = \left(\frac{8-4}{6}\right)^2 = 0.444$ for activity 1-2

Activity i-j	Activity name	t_0 (in days)	t_m (in days)	t_p (in days)	t_a (in days	σ^2
1-2	A	4	6	8	6	0.444
1-3	B	2	3	10	4	1.777
1-4	C	6	8	16	9	2.777
2-4	D	1	2	3	2	0.111
3-4	E	6	7	8	7	0.111
3-5	F	6	7	14	8	1.777
4-6	G	3	5	7	5	0.444
4-7	H	4	11	12	10	1.777
5-7	I	2	4	6	4	0.444
6-7	J	2	9	10	8	1.777

After computing the time earliest (TE) and time Latest (TL) for all the activities, below is the project network



ii) From the above network diagram Figure, the critical path is identified as 1-3,3-4, 4-6 and 6-7 with project duration of 24 days .

iii) The probability of completing the project within 19 days is given by P(Z<Z_0)

To find $Z_0 = \left(\frac{19-24}{\sqrt{1.777+0.111+0.444+1.777}}\right) = \frac{-5}{\sqrt{4.1}} = -2.469$

We know, P(Z<Z_0) Z(2.469) (from normal tables, z(2.469) = 0.9039) = 1-0.9039 = 0.0901 =9.03% Thus the probability of completing the R & D project in 19 days is 9.03%

iv) Since the probability of completing the project in 19 days is less than 20%, as in question, we find the probability of completing it in 24 days,

$Z_0 = \left(\frac{26-24}{\sqrt{2.77+0.444+1.777}}\right) = \frac{2}{\sqrt{5}} = 0.8944\ days$

$(Z \leq Z_0)$= 0.5-Z(0.8944) (from normal tables, z(1.3416) = 0.3133) = 0.5+0.3133 = 0.8133 = 81.33%;

The probability of completing it in 26 days is 81.33%

QUESTION THREE

Marking Scheme

a)	**Explanation of objective function**	1
	Explanation of constraints	1
	Maximum Marks	**2**
b)	Develop linear programming formulation	1
	Introduce slack variables	1
	Draw table 1 with all variables	1
	Calculation of Minimum of ratio	**1**
	Draw table 2 using revised basic variable	1
	Minimum of ration for the Next Iteration	1
	Draw table 3 using revised basic variable	1
	Maximum Profit	2
	Maximum Marks	**9**
c)	compute the penalty for all rows and columns	1
	Assign capacity to the highest penalty	1
	Assign capacity to the next highest penalty 4 times (1 mark each maximum 4)	4
	Initial basic feasible	1
	computation of minimum cost	2
	Maximum Marks	**9**
	Total Marks	**20**

Detailed Answer

a) An objective function represents some principal objective criterion or goal that measures the effectiveness of the system such as maximizing profits or productivity or minimizing cost or consumption. There is always some practical limitation on the availability of resources viz. man, material, machine, or time for the system. These constraints are expressed as linear equations involving the decision variables.

b) First we have to develop linear programming formulation. The linear programming formulation of the product mix problem is:

Maximize

$22x_1 + 6x_2 + 2x_3$

Subject to:

$10x_1 + 2x_2 + x_3 \leq 100$

$7x_1 + 3x_2 + 2x_3 \leq 72$

$2x_1 + 4x_2 + x_3 \leq 80$

$x_1, x_2, x_3 \geq 0$

We introduce slack variables s_4, s_5 and s_6 to make the inequalities equation.

Thus, the problem can be stated as

Maximize

$22x_1 + 6x_2 + 2x_3$

Subject to:

$10x_1 + 2x_2 + x_3 + s_4 = 100$

$7x_1 + 3x_2 + 2x_3 + s_5 = 72$

$2x_1 + 4x_2 + x_3 + s_6 = 80$

$x_1, x_2, x_3, s_4, s_5, s_6 \geq 0$

From the above equation the simplex Table 1 can be obtained in a straight forward manner.

Here is the basic variables are s_4, s_5 and s_6. Therefore $CB_1 = CB_2 = CB_3 = 0$.

CB	Basic Variable	C_j XB	22 x_1	6 x_2	2 x_3	0 s_4	0 s_5	0 s_6
0	s_4	100	10	2	1	1	0	0
0	s_5	72	7	3	2	0	1	0
0	s_6	80	2	4	1	0	0	1
	z_j-c_j		-22	-6	-2	0	0	0

Table 1

$z_1-c_1 = -22$ is the smallest negative value. Hence x_1 should be taken as a basic variable in the next iteration.

Calculate the minimum of the ratios

$$\text{Min}\left(\frac{100}{10}, \frac{72}{7}, \frac{80}{2}\right) = 10$$

The variable s_4 corresponding to which minimum occurs is made a non-basic variable.

From the Table 1, the Table 2 is calculated using the following rules:

The revised basic variables are x_1, s_5, s_6. Accordingly we make $CB_1=22$, $CB_2=0$ and $CB_3=0$. Since x_1 is the incoming variable we make x_1 coefficient one by dividing each element of row 1 by 10. Thus the numerical value of the element corresponding to x_2 is 2/10, corresponding to x_3 is 1/10, corresponding to s_4 is 1/10, corresponding to s_5 is 0/10 and corresponding to s_6 is 0/10 in Table 2.

The incoming basic variable should only appear in the first row. So we multiply first row of Table 2 by 7 and subtract if from the second row of Table 1 element by element. Thus, The element corresponding to x_1 in the second row of Table 2 is zero.

The element corresponding to x_2 is $3 - 7 * \frac{2}{10} = \frac{16}{10}$

By using this way we get the elements of the second and the third row in Table 2. Similarly, the calculation of numerical values of basic variables in Table 2 is done. By using this way we get the elements of the second and the third row in Table 2. Similarly, the calculation of numerical values of basic variables in Table 2 is done.

CB	Basic Variable	C_j XB	22 x_1	6 x_2	2 x_3	0 s_4	0 s_5	0 s_6
22	x_1	10	1	2/10	1/10	1/10	0	0
0	s_5	2	0	16/10	13/10	-7/10	1	0
0	s_6	60	0	18/5	4/5	-1/5	0	1
	z_j-c_j		0	-8/5	1/5	12/5	0	0

Table 2

$$\text{Min}\left(\frac{10}{\frac{2}{10}}, \frac{7}{\frac{16}{10}}, \frac{60}{\frac{18}{5}}\right) = \text{Min}\left(50, \frac{70}{16}, \frac{300}{18}\right) = \frac{70}{16}$$

Hence the variable s_5 will be a non-basic variable in the next iteration.

3. From Table 2, the Table 3 is calculated using the rules (i), (ii) and (iii) mentioned above.

CB	Basic Variable	C_j XB	22 x_1	6 x_2	2 x_3	0 s_4	0 s_5	0 s_6
22	x_1	73/8	1	0	-1/16	3/16	-1/8	0
6	x_2	30/8	0	1	13/16	-7/16	5/8	0
0	s_6	177/4	0	0	-17/8	11/8	-9/4	1
	z_j-c_j		0	0	24/16	24/16	1	0

Table 3

Note that all $z_j - c_j \geq 0$, so that the solution is $x_1 = 73/8$, $x_2 = 30/8$ and $s_6 = 177/4$ maximizes the objective function.

The Maximum Profit is: 22*73/8 + 6*30/8 = 1606/8 + 180/8= 1786/8 = FRW 223.25

c) Now, compute the penalty for various rows and columns which is shown in the following table:

Origin	Destination				a_i	P
	Rusizi	Huye	Rwamagana	Nyagatare		
Rubavu	20	22	17	4	120	13
Gicumbi	24	37	9	7	70	2
Kigali	32	37	20	15	50	5
b_j	60	40	30	110	240	
Row Penalty	4	15	8	3		

Look for the highest penalty in the row or column, the highest penalty occurs in the **second column** and the minimum unit cost i.e. c_{ij} in this column is c_{12}=22. Hence assign 40 to this cell i.e. x_{12}=40 and cross out the second column (since second column was satisfied. This is shown in the following table:

Origin	Destination				a_i	Column Penalty
	Rusizi	Huye	Rwamagana	Nyagatare		
Rubavu	20	22 **40**	17	4	80	13
Gicumbi	24	37	9	7	70	2
Kigali	32	37	20	15	50	5
b_j	60	40	30	110	240	
Row Penalty	4	15	8	3		

The next highest penalty in the uncrossed-out rows and columns is 13 which occur in the **first row** and the minimum unit cost in this row is c_{14}=4, hence x_{14}=80 and cross out the first row. The modified table is as follows:

Origin	Destination				a_i	P
	Rusizi	Huye	Rwamagana	Nyagatare		
Rubavu	20	22 **40**	17	4 **80**	0	13
Gicumbi	24	37	9	7	70	2
Kigali	32	37	20	15	50	5
b_j	60	40	30	110	240	
Row Penalty	4	15	8	3		

The next highest penalty in the uncrossed-out rows and columns is 8 which occurs in the **third column** and the **minimum cost** in this column is c_{23}=9, hence x_{23}=30 and cross out the third column with adjusted capacity, requirement and penalty values. The modified table is as follows:

Origin	Destination				a_i	P
	Rusizi	Huye	Rwamagana	Nyagatare		
Rubavu	20	22 **40**	17	4 **80**	0	13
Gicumbi	24	37	9 **30**	7	40	17
Kigali	32	37	20	15	50	17
b_j	60	40	30	110	240	
Row Penalty	8	15	8	8		

The next highest penalty in the uncrossed-out rows and columns is 17 which occurs in the **second row** and the **smallest cost** in this row is c_{24}=15, hence x_{24}=30 and cross out the fourth column with the adjusted capacity, requirement and penalty values. The modified table is as follows

Origin	Destination				a_i	P
	Rusizi	Huye	Rwamagana	Nyagatare		
Rubavu	20	22 **40**	17	4 **80**	0	13
Gicumbi	24	37	9 **30**	7 **30**	10	17
Kigali	32	37	20	15	50	17
b_j	60	40	30	110	240	
Row Penalty	8	15	8	8		

The next highest penalty in the uncrossed-out rows and columns is 17 which occurs in the **second row** and the **smallest cost** in this row is c_{21}=24, hence xi_{21}=10 and cross out the second row with the adjusted capacity, requirement and penalty values. The modified table is as follows:

Origin	Destination				a_i	P
	Rusizi	Huye	Rwamagana	Nyagatare		
Rubavu	20	22 **40**	17	4 **80**	0	13
Gicumbi	24 **10**	37	9 **30**	7 **30**	0	17
Kigali	32	37	20	15	50	17
b_j	60	40	30	110	240	
Row Penalty	8	15	8	8		

The next highest penalty in the uncrossed-out rows and columns is 17 which occurs in the **third row** and the **smallest cost** in this row is c_{31}=32, hence xi_{31}=50 and cross out the third row or first column. The modified table is as follows:

Origin	Destination				a_i	P
	Rusizi	Huye	Rwamagana	Nyagatare		
Rubavu	20	22 **40**	17	4 **80**	0	13
Gicumbi	24 **10**	37	9 **30**	7 **30**	0	17
Kigali	32 **50**	37	20	15	0	17
b_j	60	40	30	110	240	
Row Penalty	8	15	8	8		

Rubavu will supply Huye and Nyagatare, Gicumbi will supply Rusizi and Rwamagana whereas Kigali will supply Rusizi only. Therefore, the transportation cost corresponding to this choice of basic variables is

22x40+4x80+9x30+7x30+24x10+32x50 = FRW3520

QUESTION FOUR

Marking Guide

a)	Decision making under certainty	1
	Decision under risk	1
	Decision making under uncertainty	1
	Maximum Marks	**3**
b)	Profit formula	1
	Profit function	1
	Conditional profit values (payoff) table (0.5 Marks per each calculation Max 3)	3
	Table showing the Opportunity Loss values (0.5 Marks per each calculation Max 3)	3
	Expected opportunity cost for Develop	1
	Expected opportunity cost for not Develop	1
	Maximum Marks	**10**
C) i)	Minimum row	0.5
	Maximum column	0.5
	Comparing Max (row minima) and min(column maxima)	0.5
	Decision on saddle point	0.5
	Maximum Marks	**2**
ii)	Eliminate dominate strategy in columns	0.5
	Eliminate dominate strategy in rows	0.5
	Eliminate dominate strategy in rows for the second time	0.5
	Showing matrix in form of 2x2	0.5
	Computation of mix of strategy	2
	Value of game	1
	Maximum Marks	**5**
	Total Marks	**20**

Detailed Answer

a) The decision making under certainty, the decision-maker has complete knowledge (perfect information) of outcome due to each decision-alternative (course of action). The conditions of certainty are known and payoffs can be determined accurately. For decision under risk, decision-maker does not have perfect knowledge about possible outcome of every decision alternative. It may be due to more than one states of nature and decision making under uncertainty, decision-maker is unable to specify the probability for occurrence of state of nature .

b) If p is the proportion of customers who purchase the new product, the company's conditional profit = Marginal profit x cases sold-marginal loss x cases unsold

= (6,000-2,000) x 100p-60,000

=FRW (400,000 p – 60,000)

We have 5 possible state of nature, i.e. proportion of the customers who will buy the new product and S_1(develop the product) and S_2 (do not develop the product) be the two courses of action.

Below is the conditional profit values (payoff) table

Proportion of customers (State of Nature)	Conditional profit= FRW (400,000 p – 60,000) Course of Action	
	S_1 (Develop)	S_2 (Do not develop)
0.04	-44,000	0
0.08	-28,000	0
0.12	-12,000	0
0.16	4,000	0
0.20	20,000	0

Below is a table showing the Opportunity Loss values

Proportion of customers (State of Nature)	Probability	Conditional profit(FRW)		Opportunity Loss (FRW)	
		S_1	S_2	S_1	S_2
0.04	0.1	-44,000	0	44,000	0
0.08	0.1	-28,000	0	28,000	0
0.12	0.2	-12,000	0	12,000	0
0.16	0.4	4,000	0	0	4,000
0.20	0.2	20,000	0	0	20,000

Using the given estimates of probabilities associated with each state of nature, the expected opportunity loss (EOL) for each course of action is given below:

EOL(S_1) = 0.1(44,000)+0.1(28,000)+0.2(12,000)+0.4(0)+0.2(0) = FRW 9,600

EOL(S_2) = 0.1(0)+0.1(0)+0.2(0)+0.4(4,000)+0.2(20,000) = FRW 5,600

Since the company seeks to minimize the expected opportunity loss, the company should select course of action S_2 (do not develop the product) with minimum EOL

c) Let union proposals be Player A, proposals represented by 1, 2 and 3; manager's proposals by Player B with Contract represented by I, II, III and IV.

Player B

Player A		I	II	III	IV
	1	4	2	3	6
	2	3	4	7	5

3	6	3	5	4

i) First consider the minimum of each row

Row	Minimum value
1	2
2	3
3	3

The maximum of {2,3,3} = 3
And the maximum of each column

Column	Maximum value
I	6
II	4
III	7
IV	6

The minimum of {6,4,7,6} = 4.

The following condition holds:

Max(row minima) ≠ min(column maxima)

Therefore we see that there is no saddle point for the game under consideration

ii) Compare columns II and III.

Column II	Column III
2	3
4	7
3	5

We see that each element in column III is greater than the corresponding element in column II. The choice is for player B. Since column II dominates column III, player B will discard his strategy 3

Now we have reduced game

Player B

		I	II	IV
Player A	1	4	2	6
	2	3	4	5
	3	6	3	4

For this matrix again, there is no saddle point. Column II dominates column IV. The choice is for player B. So player B will give up his strategy 4

Now we have the reduced game again

Player B

Player A		I	II
	1	4	2
	2	3	4
	3	6	3

This matrix has no saddle point. The third row dominates the first row. The choice is for player A. He will give up his strategy 1 and retain strategy 3. The game reduces to the following:

Player B

Player A		I	II
	2	3	4
	3	6	3

Again, there is no saddle point. We have a 2x2 matrix. Take this matrix as

p = (3-6)/((3+3)-(6+4))=3/4

1-p= 1-3/4 = 1/4.

r = (3-4)/((3+3)-(6+4))=1/4

1-r= 1-1/4 = 3/4.

The Value of the game:

V= (3x3- 4x6)/(-4) = 15/4

Therefore x=(3/4,1/4,0,0) and y = (1/4,3/4,0,0) are the optimal strategies

QUESTION FIVE

Marking Guide

a)	Characteristics of Normal Probability Distribution (1 Mark per each valid characteristics Maximum 3)	3
b)	State Ho and H1 Hypothesis	1
	Calculation of Z Value	2
	Comparing sample mean and calculated value	1
	Conclusion	1
	Maximum Marks	**5**
C) i	The probability that a consumer selected at random, preferably the new brand	2
ii	The probability that a consumer preferred the new brand and was from Muhanga	2
iii	The probability that a consumer preferred the new brand, given that he was from Muhanga	2

iv	The probability that the consumer belongs to Rubavu, given that he preferred the new brand	2
	Maximum Marks	**8**
d) i)	probability that no defective	1
	Number of boxes that contain no defective	1
ii)	probability that at least 2 defectives	1
	Number of boxes that contain at least 2 defective	1
	Maximum Marks	**4**
	Total Marks	**20**

Detailed Answer

a) Two characteristics of Normal Probability Distribution

- The two tails of the normal curve extend to infinity in both directions and never touch the horizontal axis
- The mean of normal distribution may be negative, zero or positive
- The area under the normal curve represents probabilities for the normal random variable, and therefore, the total area under the curve for the normal probability distribution is 1.

b) Parameter of interest: Mean μ.

Hypotheses

$H_0 : \mu = 100.3$ min

$H_1 : \mu \neq 100.3$ min

Significance level: $\alpha = 0.01$

As the population is normally distributed and σ is known, we use the Z statistic.

We calculate $\mu_0 - z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}$ and $\mu_0 + z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}$, with n=40, and obtain 97.7546, 102.8454 respectively

As the sample mean 105.6 min does not belong to the interval [97.7546, 102.8454], we reject the null hypothesis $H_0 : \mu = 100.3$ min at the 0.01 level of significance.

Conclusion: The new battery is not more improved than the current one

c) The information during market survey is as follows:

	Nyamata	Musanze	Muhanga	Bubavu	Total
Yes	45	55	60	50	210
No	35	45	35	45	160
No opinion	5	5	5	5	20
Total	85	105	100	100	390

If X is an event that a consumer selected at random preferred the new brand

i) The probability that a consumer selected at random, preferably the new brand (X) is

$P(X)= 210/390 = 0.5398$

ii) The probability that a consumer preferred the new brand and was from Muhanga (M)

is $P(X∩M) = 60/390 = 0.1538$

iii) The probability that a consumer preferred the new brand, given that he was from Muhanga (M) is

$P(X|M) = (P(X∩C))/P(C)=(60/390)/(100/390) = 0.153/0.256 = 0.597$

iv) The probability that the consumer belongs to Rubavu, given that he preferred the new brand is

$P(M|X) = (P(M∩X))/P(X)=(50/390)/(210/390) = 0.128/0.538 = 0.237$

d) Given that p=0.1 per cent = 0.001, n= 500, $\lambda = np = 500x0.001= 0.5$

i) $P(x=0)=e^{-\lambda} = e^{-0.5}= 0.6065$

Therefore, the required number of boxes is 0.6065x100 = 61 (approx.)

ii) $P(x>2) = 1-P(x\leq 1)=1-[P(x=0)+P(x=1)]$

$= 1-[e^{\lambda} + \lambda e^{-\lambda}] = 1-[0.6065+0.5(0.6065)]$

$= 1 – 0.6065(1.5) = 1-0.90975 = 0.09025$

Therefore, the required number of boxes are 100x0.09025 = 10 (approx.)

QUESTION SIX

Marking Guide

a)	Relationship between mean, median and mode (1 Mark per each valid point Maximum 3)	3
b)	Using two remaining observations (a and b) in mean	1
	Using two remaining observations (a and b) in variance	1
	Eliminate one observation to remain with one unknow	1
	Calculating value of two remaining observation	2
	Maximum Marks	**5**
C)	Formulate equation for mean	1
	Formulate equation for frequency	1
	Use of simultaneous to find value of x and y	2
	Showing mission value	2
	Maximum Marks	**6**
D)	cumulative frequency distributions based on less than methods	2
	cumulative frequency distributions based on more than' methods	2
	Well drown ogive showing median (intersection)	2
	Maximum Marks	**6**
	Total Marks	**20**

Detailed Answer

a) The relation between mean, median and mode:

- When a distribution is symmetrical, the mean, median and mode are the same. In case, a distribution is skewed to the right, then mean> median> mode. Generally, income distribution is skewed to the right where a large number of families have relatively low income and a small number of families have extremely high income. In such a case, the mean is pulled up by the extreme high incomes and the relation among these three measures
- When a distribution is skewed to the left, then mode> median> mean. This is because here mean is pulled down below the median by extremely low values.
- Given the mean and median of a unimodal distribution, we can determine whether it is skewed to the right or left. When mean>median, it is skewed to the right; when median> mean, it is skewed to the left. It may be noted that the median is always in the middle between mean and mode

b) Let the remaining two observations be a and b, then as given

$$\frac{2+3+6+a+b}{5} = 4.80$$

11+a+b =24, therefore a+b =13 (1)

and $\frac{2^2+a^2+b^2+3^2+6^2}{5} - (4.80)^2 = 6.16$

then $49+a^2+b^2 = 146$ with $a^2+b^2 = 97$ (2)

from (1) we get a = 13-b (3)

eliminating a from (2) and (3), we get

$$(13-b)^2+b^2=97$$

then $b^2-13b+36 = 0$, by factorization (b-4)(b-9) = 0

b = 4 or 9 and from (3), a = 9 or 4

Thus the remaining observations are 4 and 9.

c) Here we are given the total number of frequencies and the arithmetic mean. We have to determine the two frequencies that are missing. Let us assume that the frequencyagainst 1 accident is x and against 2 accidents is y. If we can establish two simultaneous equations, then we can easily find the values of X and *Y*.

$$\text{Mean} = \frac{(0 \times 46) + (1 \times x) + (2 \times y) + (3 \times 25) + (4 \times 10) + (5 \times 5)}{200}$$

$$1.46 = \frac{x+2y+140}{200}$$

$x + 2y + 140 = (200)\ (1.46)$

$x + 2y = 152$ (i)

$x + y = 200 - \{46+25 + 10+5\}$

$x + y = 200 - 86$

$x + y = 114$ (ii)

Now subtracting equation (ii) from equation (i), we get

$$\begin{array}{rcl} x + 2y & = & 152 \\ x + y & = & 114 \\ - & & - \\ \hline y & = & 38 \end{array}$$

Substituting the value of $y = 38$ in equation (ii) above, $x + 38 = 114$

Therefore, $x = 114 - 38 = 76$

Hence, the missing frequencies are:

Against accident 1: 76

Against accident 2: 38

e) First of all, we transform the given data into two cumulative frequency distributions, one based on 'less than' and another on 'more than' methods.

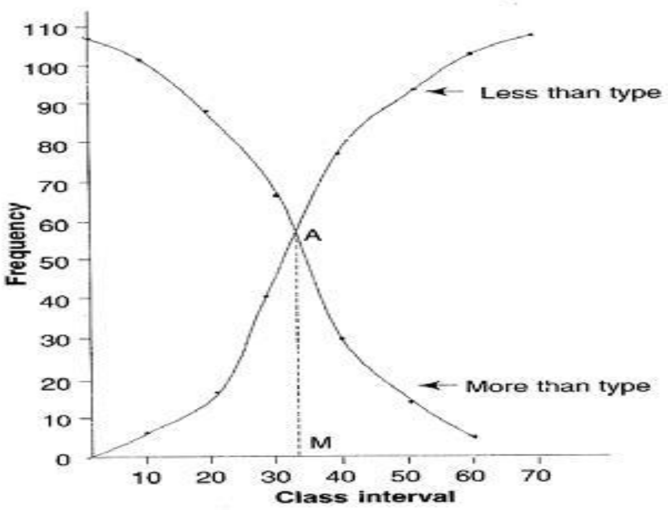
Table A

	Frequency
Less than 10	6
Less than 20	18
Less than 30	40
Less than 40	77
Less than 50	94
Less than 60	102
Less than 70	107

Table B

	Frequency
More than 0	107
More than 10	101
More than 20	89
More than 30	67
More than 40	30
More than 50	13
More than 60	5

It may be noted that the point of intersection of the two ogives gives the value of the median. From this point of intersection A, we draw a straight line to meet the X-axis at M. Thus, from the point of origin to the point at M gives the value of the median, which comes to 34, approximately.



QUESTION SEVEN

Marking Guide

a)	Computation of sample size	1
	Calculation of other required workers	1
	Maximum	**2**
b)	Calculations for Coefficient of Rank Correlation (1 mark each column maximum 6 Marks)	6
	Correlation between judge 1 and Judge 2	2
	Correlation between judge 1 and Judge 3	2
	Correlation between judge 2 and Judge 3	2
	Conclusion on judges agree most	1
	Conclusion on judges disagree most	1
	Maximum	**14**
C)	Estimation of **value of sales**	2
	Estimation of **advertisement cost**	2
	Maximum Marks	**4**
	Total Marks	**20**

Detailed Answer

a) Since $E = 2$,

$$n = \frac{z^2_{0.05/2} s^2}{E^2} = \frac{1.96^2 * 27^2}{2^2} = 700.13 \Rightarrow n = 701$$

Thus, we need ***701-81=620*** more worker**s.**

b) Calculations for Coefficient of Rank Correlation

Entry	Rank by Judges			Difference in Ranks					
	J_1	J_2	J_3	$d(J_1\&J_2)$	d^2	$d(J_1\&J_3)$	d^2	$d(J_2\&J_3)$	d^2
A	9	9	6	0	0	+3	9	+3	9
B	3	1	3	+2	4	0	0	-2	4
C	7	10	8	-3	9	-1	1	+2	4
D	5	4	7	+1	1	-2	4	-3	9
E	1	3	2	-2	4	-1	1	+1	1
F	6	8	4	-2	4	+2	4	+4	16
G	2	5	1	-3	9	+1	1	+4	16
H	4	2	5	+2	4	-1	1	-3	9
I	10	7	9	+3	9	+1	1	-2	4
J	8	6	10	+2	4	-2	4	-4	16
					$\sum d^2=48$		$\sum d^2=26$		$\sum d^2=88$

$$\rho\ (J_1\&\ J_2) = 1 - \frac{6\sum d^2}{N(N^2-1)}$$

$$= 1 - \frac{6\ x\ 48}{10(10^2-1)}$$

$$= 1 - \frac{288}{990}$$

$$= 1 - 0.29$$
$$= +0.71$$

$$\rho\ (J_1\&\ J_3) = 1 - \frac{6\sum d^2}{N(N^2-1)}$$

$$= 1 - \frac{6\ x\ 26}{10(10^2-1)}$$

$$=1 - \frac{156}{990}$$

$=1 – 0.1575$
$= +0.842$

ρ (J_2& J_3) $$=1- \frac{6\sum d^2}{N(N^2 - 1)}$$

$$=1 - \frac{6\ x\ 88}{10(10^2 - 1)}$$

$$=1 - \frac{528}{990}$$

$=1 – 0.53$
$= +0.4$

(i) Judges J_1 and J_3 agree the most
(ii) Judges J_2 and J_3 disagree the most

c) Given the following

$\overline{X}$ = FRW 25,000 S_x= FRW 25.25

$\overline{Y}$= FRW 45,000 S_y= FRW 50.50

r= 0.75

i) Using equation $Y_C - \overline{Y} = r \frac{S_y}{S_x} (X- \overline{X})$, the most appropriate value of sales Y_C for an advertisement expenditure X = FRW 50,000 is

$$Y_C – 45,000 = 0.75\ \frac{50.50}{25.25}(50,000-25,000)$$

Y_C= 45,000 + 37,500

= FRW 82,500

ii) Using equation $X_c - \bar{X} = r \frac{Sx}{Sy} (Y - \bar{Y})$, the most appropriate value advertisement expenditure X_c for achieving a sales target Y = FRW 80,000 is

$$X_c - 25,000 = 0.75 \frac{25.25}{50.50} (80,000 - 45,000)$$

$$X_c = 13,125 + 25,000$$
$$= \text{FRW } 38,125$$